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Satellite Attitude Control with Decomposed Controller

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Abstract

THE well-known decoupled torque integrator controller's configuration, which has been used for a number of years,[1-3] can be reconstructed from new point of view by using modern control theory and recommended as an unique unit for widespread use in satellite attitude control and estimation.

Contents

Most scientific and application satellites have reference attitude modes, in which one of the body axes (say x-axis) points to a fixed direction in space, while the satellite itself rotates uniformly with an angular rate ω_0 about the pointing axis. The period of the rotation may change from some tens of minutes to some hours per cycle.

To apply the reaction wheels to control the motion of such satellites is far from a trivial problem, due to the gyroscopic coupling effect in the satellite dynamic motion and the control momentum of the wheels.

The linearized coupled motion of the satellites about y- and z-axis is described by two groups of equations as follows:

1)Rotating or dynamic motion

$$I\dot{\omega}_y - a\omega_z + \dot{H}_y - \omega_0 H_z = M_y \tag{1a}$$

$$I\dot{\omega}_z + a\omega_y + \dot{H}_z + \omega_0 H_y = M_z \tag{1b}$$

where (ω_y, ω_z), (H_y, H_z) and (M_y, M_z) are the angular velocity, reaction wheel momentum, and external disturbance along appropriate axes. To simplify the analysis, we assume that the moments of intertia about y- and z-axis are identical and equal to I, and

$$a = (I - I_x)\omega_0 - H_x$$

2) Pointing or kinematic coupling motion

$$\dot{f}_y - \omega_0 f_z = \omega_y \qquad \dot{f}_z + \omega_0 f_y = \omega_z \tag{2}$$

where the f_y, f_z are the attitude pointing deviation angles of x-axis from its reference.

Introducing equivalent control torques u_1 and u_2 defined as

$$u_1 = \dot{H}_y - \omega_0 H_z \qquad u_2 = \dot{H}_z + \omega_0 H_y \tag{3}$$

and the state vector and control vector

$$X = \begin{bmatrix} \omega_y \\ \omega_z \end{bmatrix} \qquad u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

and corresponding matrices

$$A = \begin{bmatrix} 0 & a/I \\ -a/I & 0 \end{bmatrix} \quad B = \begin{bmatrix} -1/I & 0 \\ 0 & -1/I \end{bmatrix}$$

an optimal decoupling control problem of Eqs. (1) can be defined as follows.

Given

$$\dot{X} = AX + Bu \tag{4}$$

find the control vector u so that the cost function

$$J = X'(T)FX(T) + \int_0^T (X'QX + u'Ru)\,dt \tag{5}$$

is a minimum. The F, Q are semipositive definite, and R is a positive definite symmetric matrices.

This control problem has solution in the form[4]

$$u = -R^{-1}B'KX \tag{6}$$

$$\dot{K} = -KA - A'K - Q + KBR^{-1}B'K \qquad K(T) = F \tag{7}$$

When the Q and R are chosen to be diagonal matrices with elements q=const and r=const, then the steady solution of the problem has the elements of K in the form

$$k_{11} = k_{22} = I(qr)^{1/2} \qquad k_{12} = k_{21} = 0$$

and from Eqs. (3) and (6) it follows that

$$u_1 = \dot{H}_y - \omega_0 H_z = k\omega_y \qquad u_2 = \dot{H}_z + \omega_0 H_y = k\omega_z$$
$$k = (q/r)^{1/2} \tag{8}$$

Equation (8) shows that the solution of the optimal decoupling control is a feedback law in the wheel equivalent control torques of ω_y and ω_z.

Now it is necessary to find a way to implement this feedback law. In achieving this aim, a control concept called Decomposed Controller (DC) will be introduced. The DC consists of two coupled electronic integrators,[1-3] the inputs of which are the RG measured quantities $k_1\omega_y$ and $k_1\omega_z$, and the electrical outputs are U_y and U_z in volts. The DC must imitate the optimal control law, Eq. (8), and adjust the wheel momentum changes (see Fig. 1)

$$\dot{U}_y - \omega_0 U_z = k_1\omega_y \qquad \dot{U}_z + \omega_0 U_y = k_1\omega_z \tag{9}$$

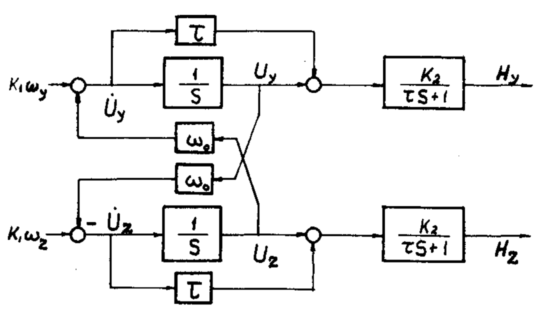

Fig. 1 DC-wheel control system block diagram.

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By comparing Eqs. (8) and (9), it can be seen that if the wheel momentum H_y and H_z track perfectly the k_2 times DC outputs k_2U_y and k_2U_z (for $k_1k_2=k$), then the optimal decoupling control obviously takes place and Eq. (4) is strictly stable.

Now let us solve the tracking problem. Suppose the wheel is characterized by first order dynamic behavior, and the voltages u_y and u_z are applied to the input terminals

$$\tau\dot{H}_y+H_y=k_2u_y \qquad \tau\dot{H}_z+H_z=k_2u_z \tag{10a}$$

By introducing new variables

$$e_y=H_y-k_2U_y \qquad v_y=u_y-(\tau\dot{U}_y+U_y) \tag{10b}$$

then the left side of Eq. (10a) can be rewritten as follows

$$\dot{e}_y=-e_y/\tau+k_2v_y/\tau \tag{11}$$

An optimal tracking problem for Eq. (11) can be formulated in the same way as for the optimizing system of Eq. (4). One difference is including the term $q_2\dot{e}_y^2$ in the cost function to depress the dynamic tracking error

$$J=\int_0^\infty(q_1e_y^2+q_2\dot{e}_y^2+r_1v_y^2)\,dt \tag{12}$$

Using Eq. (11), we can write

$$q_2\dot{e}_y^2=q'e_y^2+2le_yv_y+r'v_y^2$$

Then the cost function Eq. (12) can be transformed into

$$J=\int_0^\infty[e_y \quad v_y]\begin{bmatrix} q & l \\ l & r\end{bmatrix}\begin{bmatrix} e_y \\ V_y\end{bmatrix}dt \tag{13}$$

where $q=q_1+q'$, $r=r_1+r'$, and the matrix must be positive definite. Then the optimal tracking problem can be defined as follows: to find control v_y so that the cost function Eq. (13) is minimized.

The solution of the problem is[4]

$$v_y=-ge_y \qquad g=r^{-1}(l+k_2K/\tau)$$

$$2K/\tau-q+(l+k_2K/\tau)^2r^{-1}=0$$

and Eq. (11) becomes

$$\tau\dot{e}_y+e_y=-gk_2e_y \tag{14a}$$

Similarly,

$$\tau\dot{e}_z+e_z=-gk_2e_z \tag{14b}$$

or, after substituting e_y and e_z by the expression of Eq. (10b),

$$\begin{aligned}\tau\dot{H}_y+H_y&=k_2(\tau\dot{U}_y+U_y)-gk_2(H_y-k_2U_y)\\ \tau\dot{H}_z+H_z&=k_2(\tau\dot{U}_z+U_z)-gk_2(H_z-k_2U_z)\end{aligned} \tag{15}$$

The computer simulation shows that the tracking performance remains very good even for $g=0$. In this case, the wheel tracking loop structure can be further simplified, and the block diagram of DC-wheel optimal decoupling control concept is presented in Fig. 1.

The key point in obtaining the simple structure of the control system without losing its performance, is that the defined DC in Eqs. (9) provides a possibility of decomposing the optimal control problem of the satellite-wheel composite system of Eq. (1), which is a very complex one, into two subproblems:

1) finding the optimal decoupling (or nutation damping) control law Eqs. (8); and

2) realizing the law that is the optimal wheel tracking control Eqs. (15).

Of course, we could formulate other optimal control problems by modifying the system Eqs. (4) and (5), and the optimal control law obtained would be different than that of Eqs. (8). However, the only change is in the input of DC, and the realization of the new optimal control law should be the same as Eq. (15). Therefore, the DC is an independent and unique unit, which simplifies the control problem formulation and the structure of the overall control system immensely.

Multiply both sides in Eq. (2) by k_1

$$k_1\dot{f}_y-\omega_0k_1f_z=k_1\omega_y \qquad k_1\dot{f}_z+\omega_0k_1f_y=k_1\omega_z \tag{16}$$

Equation (16) has the same form as Eq. (9). Consequently, the DC with initial values $U_y(t_0)$ and $U_z(t_0)$ equal to $k_1f_y(t_0)$ and $k_1f_z(t_0)$, respectively, represents not only the optimal control law, but also the evolution of f_y and f_z via the relations

$$\begin{aligned}f_y(t)&=U_y(t)/k_1=H_y(t)/k\\ f_z(t)&=U_z(t)/k_1=H_z(t)/k\end{aligned} \tag{17}$$

Therefore, the DC can be used as a model for the attitude estimation in some cases.

The synchronous behavior of the triplet (U,H,f) motivates a very simple method of attitude pointing control, i.e., every time the wheel momentum is being discharged, the attitude pointing error angle reduces to zero automatically and monotonically, without overshoot. The maximum pointing error is determined as follows:

$$f_{y\max}=H_{y\max}/k \qquad f_{z\max}=H_{z\max}/k \tag{18}$$

Therefore, by choosing appropriate gain factor k or discharging value $H_{\max}$ or both, the maxium pointing error does not exceed a designed and desired value.

For other details about the DC concept, readers should refer to the backup paper.

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